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PRODUCTION AND INDUSTRIAL ENGINEERING

Objective Practice Sets

Engineering Mathematics

Contents

MADE EASY PUBLICATIONS

1 CHAPTER

Linear Algebra

Q.1 Determine the rank of the following matrix :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \\ 3 & 1 & -3 & -1 \\ 1 & 1 & -3 & -1 \end{bmatrix}$$

Q.2 The eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

(a) (4, 1) and (1, –1) (b) (0, 1) and (1, –1)
(c) (4, 0) and (1, 0) (d) (4, 1) and (0, –1)

Q.3 The eigen vector of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ corresponding to smallest eigen value is

(a) (3, 2, 1) (b) (1, 0, 0)
(c) (1, –1, 0) (d) None

Q.4 By using Cayley Hamilton theorem, if $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, then express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

(a) $A + 5I$ (b) $2A + 5I$
(c) $A + 7I$ (d) $2A + 7I$

Q.5 The determinant of the matrix A $\begin{vmatrix} 6 & 2 & 3 \\ 2 & 3 & 5 \\ 4 & 2 & 1 \end{vmatrix}$ is

Q.6 The value of the determinant $\begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix}$ is

(a) 0 (b) – 1
(c) 1 (d) 2

Q.7 Eigen values of $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ are

(a) – 6, – 1 (b) 6, – 1
(c) – 6, 1 (d) 6, 1

Q.8 The Algebraic multiplicity of the matrix $A = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{vmatrix}$ is

(a) 1 (b) 2
(c) 3 (d) 4

Q.9 The matrix $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & x \end{bmatrix}$ is its own adjoint. The value of x will be

(a) 5 (b) 3
(c) –3 (d) –5

Q.10 The system of equations $x + y + z = 6$, $2x + y + z = 7$, $x + 2y + z = 8$ has

(a) A unique solution
(b) No solution
(c) An infinite number of solutions
(d) None of these

Q.11 If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(a) $\alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 + \beta\gamma = 0$
(c) $1 - \alpha^2 - \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$

Q.12 All four entries of 2 × 2 matrix $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$ are non-zero, and one of its eigen value is zero. Which one of the following statement is true?

(a) $B_{11} B_{22} - B_{12} B_{21} = 1$
(b) $B_{11} B_{22} - B_{12} B_{21} = -1$
(c) $B_{11} B_{22} - B_{12} B_{21} = 0$
(d) $B_{11} B_{22} + B_{12} B_{21} = 0$

Q.13 Which of the following statements are TRUE?
1. The eigen values of a Hermitian matrix are real.
2. The value of the determinant of an orthogonal matrix can only be +1.
3. The transpose of a square matrix A has the same eigen values as those of A.
4. The inverse of '$n \times n$' matrix exists if and only if the rank is less than 'n'.

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1 and 4 only

Q.14 Let matrix $[A]_{m \times n}$ has rank a and matrix $[B]_{n \times p}$ has rank b. Then matrix $[AB]$ may have rank
(a) $\leq \max(a, b)$ (b) n
(c) $a + b$ (d) $\leq \min(a, b)$

Q.15 Let the eigen values of matrix $[A]_{2 \times 2}$ are α and β then eigen values for $(A + 5I)^{-1}$ are
(a) $(\alpha + 5), (\beta + 5)$ (b) $\frac{1}{\alpha + 5}, \frac{1}{\beta + 5}$
(c) $\frac{1}{\alpha} + 5, \frac{1}{\beta} + 5$ (d) can't be determined

Q.16 Given that: $A = \begin{bmatrix} -5 & -2 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is
(a) $19A + 20I$ (b) $19A + 30I$
(c) $21A + 20I$ (d) $21A + 30I$

Q.17 The determinant of the given matrix is ___________.

$$M = \begin{bmatrix} 0 & 0 & 0 & 3 & 0 \\ -2 & 0 & 0 & 2 & 0 \\ 8 & -1 & 0 & -7 & 2 \\ -1 & 2 & 2 & 3 & 2 \\ 2 & 2 & 3 & 6 & 4 \end{bmatrix}$$

Q.18 If A is an orthogonal matrix then
(a) $[A]^T = [A]^{-1}$ (b) $|A| = \pm 1$
(c) $[A] \times [A]^T = [I]^2$ (d) All of these

Q.19 If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then eigen values for matrix A are
(a) $\pm \cos\alpha$ (b) $\pm \sin\alpha$
(c) $\cos\alpha \pm \sin\alpha$ (d) $e^{\pm i\alpha}$

Q.20 In the matrix $[A] = \begin{bmatrix} \sin\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ the number of non-zero co-factors of matrix $[A]$ is ______.

Q.21 If x, y, z are in Arithmetic Progression (AP) with common difference 'd' and the rank of the matrix $[A]$ is 2.

$$[A] = \begin{bmatrix} 4 & 5 & x \\ 5 & 6 & y \\ 6 & k & z \end{bmatrix}$$

The values of k and d are respectively.
(a) $k = 7$ and d can be any integer
(b) k = multiple of 7 and d can be any integer
(c) k = any multiple of 7, $d = \frac{x}{4}$
(d) $k = 7$, $d = \frac{x}{4}$

Q.22 If the system of equations

$$ax + by + c = 0$$
$$bx + cy + a = 0$$
$$cx + ay + b = 0$$

has a unique solution then system of equations

$(a + b)x + (b + c)y + (c + a) = 0$
$(b + c)x + (c + a)y + (a + b) = 0$
$(c + a)x + (a + b)y + (b + c) = 0$

has
(a) only one solution
(b) no solution
(c) infinite number of solutions
(d) can't be determined

Q.23 Consider the 2×2 matrix $\begin{bmatrix} 1 & 2 \\ p & 5 \end{bmatrix}$. The range of possible values of p, for which both the eigen values of the matrix are real and positive, is
(a) $-\frac{5}{2} \leq p \leq \frac{5}{2}$ (b) $2 \leq p \leq \frac{5}{2}$
(c) $-2 \leq p \leq \frac{5}{2}$ (d) $-\frac{5}{2} \leq p \leq 2$

Answers Linear Algebra

1. (2)	2. (a)	3. (c)	4. (a)	5. (–30)	6. (c)	7. (a)	8. (a)	9. (b)
10. (a)	11. (c)	12. (c)	13. (b)	14. (d)	15. (b)	16. (c)	17. (12)	18. (d)
19. (d)	20. (5)	21. (d)	22. (a)	23. (c)	24. (b)	25. (d)	26. (–15)	27. (b)
28. (–22)	29. (d)	30. (9)	31. (b)	32. (b)	33. (a)	34. (6)	35. (c)	

Explanations Linear Algebra

1. (2)

Given matrix

$$\sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \\ 3 & 1 & -3 & -1 \\ 1 & 1 & -3 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{(Operating } R_3 - R_1,\ R_4 - R_1\text{)}$$

$$\sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{(Operating } R_3 - 3R_2,\ R_4 - R_2\text{)}$$

(Operating $C_3 + 3C_2$, $C_4 + C_2$)

$$\sim \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A \text{ (say)}$$

Obviously, the 4th order minor of matrix is zero. Also every 3rd order minor of A is zero. But, of all the 2nd order minors, only $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = -1 \neq 0.$

$\therefore \quad \rho(A) = 2$

Hence, the rank of the given matrix is 2.

2. (a)

The characteristic equation is $[A - \lambda I] = 0$

i.e., $\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$

or $\lambda^2 - 7\lambda + 6 = 0$

or $(\lambda - 6)(\lambda - 1) = 0$

$\therefore \quad \lambda = 6, 1$

Thus, the eigen values are 6 and 1.

If x, y be the components of an eigen vector corresponding to the eigen value λ, then

$$[A - \lambda I]X = \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Corresponding to $\lambda = 6$, we have

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

which gives only one independent equation $-x + 4y = 0$.

$\therefore \ \frac{x}{4} = \frac{y}{1}$ giving the eigen vector (4, 1).

Corresponding to $\lambda = 1$, we have

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

which gives only one independent equation $x + y = 0$.

$\therefore \ \frac{x}{1} = \frac{y}{-1}$ giving the eigen vector (1, –1).

3. (c)

The characteristic equation is

$$[A - \lambda I] = 0,$$

i.e., $\begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} = 0$

or $\quad (3-\lambda)(2-\lambda)(5-\lambda) = 0$

Thus, the eigen values of A are 2, 3, 5.

If x, y, z be the components of an eigen vector corresponding to the eigen value λ, we have

$$[A - \lambda I]X = \begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$